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A STUDY OF BUCKLING AND VIBRATION OF LAMINATED SHALLOW CURVED PANELS

IZHAK SHEINMAN and YOSI REICHMAN
Faculty of Civil Engineering, Technion–Israel Institute of Technology, Technion City, Haifa 32 000, Israel

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Abstract—An analytical tool for buckling and vibration analysis of laminated shallow curved panels is presented. The equations in terms of transverse displacement and Airy stress function are derived via the Hu–Washizu mixed formulation and solved by the Ritz method, using the eigenfunctions of an isotropic beam. The effect of the prebuckling state and out-of-plane natural boundary conditions is examined. The approximate reduced bending stiffness method is evaluated for cross-ply and angle-ply laminates.

INTRODUCTION

High-performance composite materials are in extensive use in laminated thin walled structures subjected to compression as well as to tension. Consequently, both relevant modes of behavior—buckling and vibration—have to be considered. These modes are in fact a routine source of insight into the response of these structures, with its vital safety aspect, especially in the case of curved panels. Thus, improved behavior prediction accuracy is essential for reliable design.

Many research works on buckling and vibration analysis of various types of structures have been reported in the literature. Extensive reviews by Leissa (1978, 1985) and by Bert (1979, 1982) indicate that most of the research has been confined to flat panels, while very few works have dealt with cylindrical panel behavior, the latter being both theoretical (Crawley, 1979; Zhany and Matthews, 1983; Whitney, 1983; Baharlou, 1985; Tennyson, 1986) and experimental (Wilkins, 1975; Becker *et al.*, 1982; Bauld and Khot, 1982; Kobayashi *et al.*, 1986). Since curved panels are characterized by a limit point rather than by a bifurcation point, many of the works are concerned with the postbuckling behavior [see Sheinman and Frostig (1990)].

One of the most important parameters of composite laminated structures is the stretching–bending coupling effect, research work in which has been reviewed by Kicher and Mandell (1971) for flat panels. The pioneer work in this context was that of Reissner and Stavsky (1961), followed by that of Whitney and Leissa (1969) for cross-ply and angle-ply laminates and also that of Chamis (1969). Ashton (1969) proposed an approximate solution method called RBS (Reduced Bending Stiffness), based on the decoupling of the stress function and the transverse displacements in the potential energy.

The aim of the present work is to develop an analytical tool for buckling and vibration frequencies for further investigating the approximate RBS method and to study the effect of curvature on this method. Another important parameter which is usually neglected is the prebuckling state. The present work deals with the effect of the prebuckling state (assuming out-of-plane prebuckling displacement due to compliance with the natural boundary conditions and nonconstant stress behavior during buckling) on the buckling and vibration–buckling interaction curves.

The panel may consist of curvatures in the two principal directions, longitudinal and lateral (see Fig. 1). The analysis uses the Von Karman kinematic approach and the classical elastic laminate principles. The equations, in terms of transverse displacement and the Airy stress function, are derived via the Hu–Washizu mixed formulation of the potential energy. The solution procedure is based on the Ritz method for which the dependent variables are separated into the eigenfunctions of an isotropic beam. Lagrange multipliers are introduced

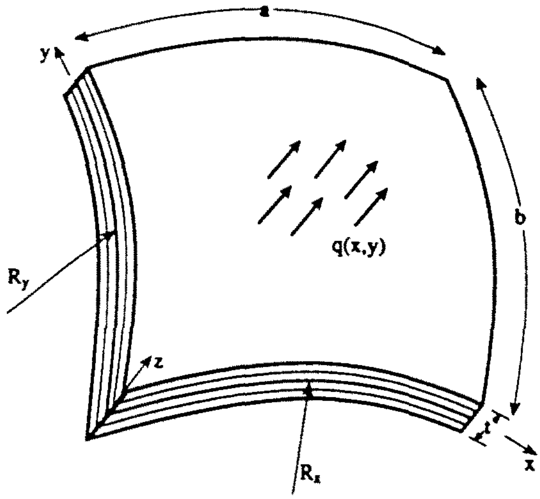

Fig. 1. Geometry.

to the potential energy to satisfy the natural out-of-plane boundary conditions. Examples of cross-ply and angle-ply laminates are used for illustration.

ANALYTICAL FORMULATION

Consider a composite shallow curved panel consisting of homogeneous orthotropic layers of arbitrary orientation and combination, with total thickness t. Let (x, y) be the panel coordinates of the reference surface, z the normal coordinate, R_x and R_y the radii of curvature in the x- and y-directions (Fig. 1). Recourse to the Kirchhoff-Love hypothesis leaves only three dependent variables, namely the displacements u, v, w in the x-, y- and z-directions, respectively. Resorting to the Von Karman kinematic approach, the strain-displacement relation can be written as:

$$\{\varepsilon\} = \{\bar{\varepsilon}\} + z\{\chi\} \tag{1}$$

where

$$\{\bar{\varepsilon}\} = \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\gamma}_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} + \frac{1}{2}w_{,x}^2 + \dfrac{w}{R_x} \\ v_{,y} + \frac{1}{2}w_{,y}^2 + \dfrac{w}{R_y} \\ u_{,y} + v_{,x} + w_{,x}w_{,y} \end{Bmatrix}$$

$$\{\chi\} = \{\chi_{xx} \quad \chi_{yy} \quad 2\chi_{xy}\} = \{-w_{,xx} \quad w_{,yy} \quad -2w_{,xy}\} \tag{2}$$

$(\ )_{,x}$ and $(\ )_{,y}$ denote the derivatives with respect to x and y, respectively.

Under the classical laminate theory, the strain $\{\bar{\varepsilon}\}$ and the bending moment $\{M\}$ (M_{xx}, M_{yy}, M_{xy}) vectors can be expressed in terms of the Airy stress function $\{F\}$ $(\{F_{,yy}, F_{,xx}, -F_{,xy}\} = \{N_{xx}, N_{yy}, N_{xy}\})$ and curvature $\{\chi\}$ vectors as [see Sheinman and Frostig (1988)]:

$$\begin{aligned} \{\bar{\varepsilon}\} &= [a]\{F\} - [b]\{\chi\} \\ \{M\} &= [b]^T\{F\} + [d]\{\chi\} \end{aligned} \tag{3}$$

where

$$a = A^{-1},\quad b = A^{-1}B,\quad d = D - BA^{-1}B$$

$$(A_{ij}\quad B_{ij}\quad D_{ij}) = \int_z Q_{ij}(1\quad z\quad z^2)\,\mathrm{d}z, \tag{4}$$

A_{ij}, B_{ij} and D_{ij} being, respectively, the membrane, coupling and flexural rigidities, and Q_{ij} the laminate transformation reduced stiffnesses.

The governing equations in terms of the transverse displacement (w) and the Airy stress function (F) are derived via the Hu–Washizu mixed formulation of the potential energy:

$$\pi = \int_x \int_y \Big\{ \tfrac{1}{2}\Big(-\{F\}^{\mathrm{T}}[a]\{F\} + 2\{F\}^{\mathrm{T}}[b]\{\chi\} + \{\chi\}^{\mathrm{T}}[d]\{\chi\} + F_{,yy}w_{,x}^2 + F_{,xx}w_{,y}^2 - 2F_{,xy}w_{,x}w_{,y} + 2F_{,yy}\frac{w}{R_x} + 2F_{,xx}\frac{w}{R_y}\Big) - qw \Big\}\,\mathrm{d}x\,\mathrm{d}y \tag{5}$$

where q is the external applied normal load.

This mixed expression [the one given by Ashton (1969) is incorrect] includes coupling of the Airy stress vector $\{F\}$ and the change of curvature vector $\{\chi\}$. Variation of π yields the exact equilibrium and compatibility equations [see Sheinman and Frostig (1990)].

Employing the perturbation technique:

$$\begin{aligned} w &= w^{(0)} + \lambda w^{(1)} \\ F &= F^{(0)} + \lambda F^{(1)} \end{aligned} \tag{6}$$

yields the following expression for the potential energy:

$$\pi = \pi^{(0)} + \lambda\pi^{(1)} + \lambda^2\pi^{(2)} \tag{7}$$

where

$$\begin{aligned} \pi^{(1)} = \int_x \int_y \{ \tfrac{1}{2}(&-2\{F^{(0)}\}^{\mathrm{T}}[a]\{F^{(1)}\} + 2\{F^{(0)}\}^{\mathrm{T}}[b]\{\chi^{(1)}\} \\ &+ 2\{F^{(1)}\}^{\mathrm{T}}[b]\{\chi^{(0)}\} + 2\{\chi^{(0)}\}[d]\{\chi^{(1)}\} + 2F^{(0)}_{,yy}w^{(0)}_{,x}w^{(1)}_{,x} \\ &+ 2F^{(0)}_{,xx}w^{(0)}_{,y}w^{(1)}_{,y} - 2F^{(0)}_{,xy}(w^{(0)}_{,x}w^{(1)}_{,y} + w^{(1)}_{,x}w^{(0)}_{,y}) + F^{(1)}_{,yy}w^{(0)^2}_{,x} \\ &+ F^{(0)}_{,xx}w^{(0)^2}_{,y} - 2F^{(1)}_{,xy}w^{(0)}_{,x}w^{(1)}_{,y}) - qw^{(1)} \}\,\mathrm{d}x\,\mathrm{d}y. \end{aligned} \tag{8}$$

$\pi^{(2)}$ can be decomposed as:

$$\pi^{(2)} = \pi_{\mathrm{L}} + \pi^{(1)}_{\mathrm{NL}} + \pi^{(2)}_{\mathrm{NL}} \tag{9}$$

where

$$\pi_{\mathrm{L}} = \iint_{xy} \Big\{ \tfrac{1}{2}\Big(-\{F^{(1)}\}^{\mathrm{T}}[a]\{F^{(1)}\} + 2\{F^{(1)}\}^{\mathrm{T}}[b]\{F^{(1)}\} + \{\chi^{(1)}\}[d]\{\chi^{(1)}\} + 2F^{(1)}_{,yy}\frac{w^{(1)}}{R_x} + 2F^{(1)}_{,xx}\frac{w^{(1)}}{R_y}\Big)\Big\}\,\mathrm{d}x\,\mathrm{d}y$$

$$\pi_{NL}^{(1)} = \iint_{xy} \{\tfrac{1}{2}(\bar{N}_{xx} w_{,x}^{(1)2} + \bar{N}_{yy} w_{,y}^{(1)2} - 2\bar{N}_{xy} w_{,x}^{(1)} w_{,y}^{(1)})\} \, dx \, dy$$

$$\times \xi_f \iint_{xy} \{\tfrac{1}{2}(F_{,yy}^{(0)} w_{,x}^{(1)2} + F_{,xx}^{(0)} w_{,y}^{(1)2} - 2F_{,xy}^{(0)} w_{,x}^{(1)} w_{,y}^{(1)})\} \, dx \, dy$$

$$\pi_{NL}^{(2)} = \xi_w \iint_{xy} \{\tfrac{1}{2}(2F_{,yy}^{(1)} w_{,x}^{(0)} w_{,x}^{(1)} + 2F_{,xx}^{(1)} w_{,y}^{(0)} w_{,y}^{(1)} - 2F_{,xy}^{(1)}(w_{,x}^{(0)} w_{,y}^{(1)} + w_{,x}^{(1)} w_{,y}^{(0)}))\} \, dx \, dy. \tag{10}$$

$\bar{N}_{xx}$, $\bar{N}_{yy}$ and $\bar{N}_{xy}$ are the in-plane external loads in the x- and y-directions and the external in-plane shear loading respectively, applied at the boundaries. The prebuckling and buckling equations are derived by setting $\delta\pi^{(1)} = 0$ and $\delta\pi^{(2)} = 0$ respectively. The tracer parameters ξ_f and ξ_w (with values 0 or 1) represent the following situations:

(i) $\xi_f = 0$ constant inplane internal forces.
(ii) $\xi_f = 1$ the general Airy function depends on the panel coordinate system.
(iii) $\xi_w = 0$ the assumption that the prebuckling response is pure membrane ($w^{(0)} = 0$).

By introducing the kinematic energy

$$T = \frac{1}{2}\iint_{xy} \rho w_{,t}^2 \, dx \, dy \tag{11}$$

the vibration domain is included.

$w^{(0)}$ and $F^{(0)}$, which are obtained from the prebuckling state, are governed by the external loading, boundary conditions and radii of curvature.

Solution procedure

The displacement and Airy stress functions assume the following form:

$$w(x, y, t) = e^{i\omega t} \sum_m^{mw} \sum_n^{nw} w_{mn} s_m(x) t_n(y)$$
$$F(x, y, t) = e^{i\omega t} \sum_m^{mf} \sum_n^{nf} f_{mn} g_m(x) r_n(y), \tag{12}$$

ω being the free vibration frequency, mw, nw, mf and nf the number of terms in the truncated series for w and F, respectively. The displacement functions $s_m(x)$, $t_n(y)$ have to satisfy the geometric boundary conditions, the stress functions $g_m(x)$, $r_n(y)$ taking the form of a clamped–clamped isotropic beam mode and satisfying the in-plane force conditions at the boundaries. The most commonly used $s(x)$, $t(y)$, $g(x)$, $r(y)$ are derived either from the beam vibration mode:

$$\phi(\ ) = C_1 \sin \beta_m(\ ) + C_2 \cos \beta_m(\ ) + C_3 \sinh \beta_m(\ ) + C_4 \cosh \beta_m(\ ) \tag{13}$$

or from the column buckling mode:

$$\phi(\ ) = C_1 \sin \beta_m(\ ) + C_2 \cos \beta_m(\ ) + C_3 + C_4(\ ). \tag{14}$$

The coefficients C_i are determined through the end conditions, while the terms β_m are eigenvalues obtained from the characteristic equations [see Sheinman *et al.* (1991)].

Compliance with the out-of-plane natural boundary conditions is provided by recourse to the Lagrange multipliers and inclusion of the following expression in the potential energy:

$$\pi_B = \xi_{vl} \oint \{M_{xx}(0,y)w(0,y)_{,x} - M_{xx}(a,y)w(a,y)_{,x} + Q_{xx}(0,y)w(0,y) - Q_{xx}(a,y)w(a,y)\}\,\mathrm{d}y + \{M_{yy}(x,0)w(x,0)_{,y} - M_{yy}(x,b)w(x,b)_{,y} + Q_{yy}(x,0)w(x,0) - Q_{xx}(x,b)w(x,b)\}\,\mathrm{d}x, \quad (15)$$

ξ_{vl} again being a tracer (0 or 1) representing compliance with the natural out-of-plane boundary conditions for the buckling and frequency of the panel.

Substituting the isotropic-beam eigenmodes [eqn (13) or (14)] in the displacement and Airy functions, integrating in both the x- and y-directions (with the aid of a symbolic compiler) and applying the variational principle on $\pi^{(2)}$, we obtain the following matrix equation:

$$\left\{\left[\begin{array}{c|c} K_{ff} & K_{fw} \\ \hline K_{fw}^{T} & K_{ww} \end{array}\right] - \lambda\left[\begin{array}{c|c} & G_{fw} \\ \hline G_{fw}^{T} & G_{ww} \end{array}\right] - \omega^2\left[\begin{array}{c|c} & \\ \hline & M_{ww} \end{array}\right]\right\}\begin{Bmatrix} f \\ w \end{Bmatrix} = \{0\}. \quad (16)$$

This equation is an eigenvalue problem for which λ represents the buckling load parameter and ω the free vibration frequency. The effect of the prebuckling state on the frequency under a given loading, can be also treated through this equation. K is the matrix (mixed stiffness and flexibility, nonpositive definite) derived via $\delta\pi_L = 0$; K_{ff} consists of the in-plane coefficients a_{ij}, K_{ww} of the flexural coefficients d_{ij} and K_{fw} of the coupling coefficients b_{ij}; K_{fw} contains also terms due to the curvature. The RBS method consists in setting $b_{ij} = 0$ in the K_{fw} matrix—equivalent to omission of the coupling terms $[b]\{F\}$ in the equilibrium equation and $[b]\{\chi\}$ in the compatibility ones. G is the so-called geometric matrix derived from $\delta\pi_{NL} = 0$; G_{fw} contains the prebuckling transverse displacement $w^{(0)}_{mn}$, and G_{ww} the prebuckling Airy function $f^{(0)}_{mn}$, which is usually not considered [see for example Baharlou and Leissa (1987)]. Since the mass matrix M is defined only for the w terms, the effect of the b_{ij} coefficients in the K_{fw} matrix is insignificant in frequency analysis, which enhanced the popularity of the RBS method. By contrast, in buckling analysis, where G_{fw} does not vanish, the effect of the b_{ij} terms may be pronounced. G_{fw} is mainly affected by the prebuckling transverse displacement $w^{(0)}$, hence the procedure sets out from the prebuckling state:

$$\begin{bmatrix} K_{ff} & K_{fw} \\ K_{fw}^{T} & K_{ww} \end{bmatrix}\begin{Bmatrix} f^{(0)} \\ w^{(0)} \end{Bmatrix} = \{p\} \quad (17)$$

where $\{p\}$ contains the external applied load as well as the natural out-of-plane boundary condition due to $\delta\pi_B$. Thus, the $w^{(0)}$ and $f^{(0)}$ vectors are obtained from eqn (17) and substituted in the geometric matrix G.

Numerical results and discussion

The procedure outlined in the preceding section is used to study the effect of some parameters on buckling and vibration of laminated curved panels in the context of the RBS (Reduced Bending Stiffness) method. For this purpose, a rectangular simply-supported curved panel was taken with data as follows [see Sheinman and Frostig (1990)]: 2-ply carbon epoxy laminate with $E_{11} = 2.07 \cdot 10^{11}$ N m^{-2}, $E_{22} = 5.2 \cdot 10^9$ N m^{-2}, $G_{12} = 2.7 \cdot 10^9$ N m^{-2}, $\nu_{12} = 0.25$, thickness $h = 2.5$ mm, length $a = 0.25$ m, width $b = 0.25$ m and mass density $\rho = 1600$ kg m^{-3}. Two well-known layup cases (a) cross-ply and (b) angle-ply were considered as illustrations.

(a) *Cross-ply laminate*. For the cross-ply laminate, the difference between the approximate RBS solution and its complete counterpart increases with the aspect ratio of the panel [see Whitney and Leissa (1969), Kicher and Mandell (1971)] and is very small for a rectangular panel. However, cross-ply laminates are not symmetric with respect to the middle surface and the contribution of fiber orientation in the outer and inner layers must

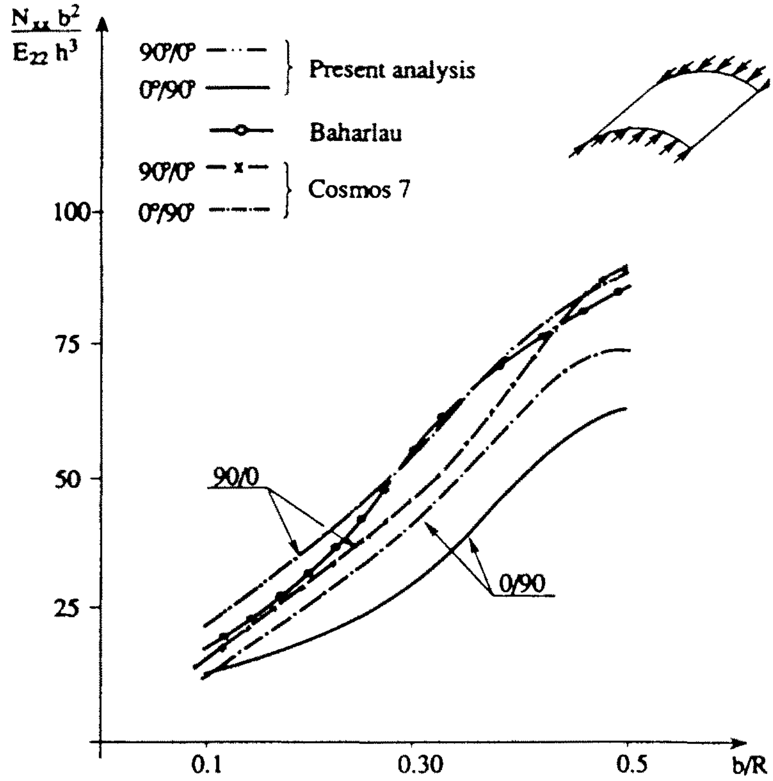

Fig. 2. Axial buckling load of cross-ply laminate versus radius of curvature.

be taken into account when studying the effect of curvature. To this end, the buckling load of cross-ply laminates with (90,0) and (0,90) layups, under axial compression was examined for several values of curvature. The nondimensional results are plotted against the curvature parameter in Fig. 2 and listed in Table 1. The present analysis yielded a significant difference between the two layup variants. This difference is largely attributable to the ξ_M parameter (see Table 1), which is introduced in order to satisfy the out-of-plane natural boundary conditions, and at the value $\xi_M = 1$ yields a nonmembrane prebuckling behavior. The small difference observed in the earlier study by Baharlou (1985), is obviously due to disregard

Table 1. Buckling load of curved cross-ply laminate panel with simply-supported boundary conditions

		$N_{xx}b^2/E_{22}h^3$							
						Present analysis			COSMOS7
b/R_x	Layup	$\xi_l = 0$	$\xi_w = 0$	$\xi_l = \xi_w = 0$	$\xi_M = 0$	RBS	Complete	Baharlou (1985)	Lashkari (1984)
0	90/0	13.33	12.67	12.66	12.65	13.32	13.34	12.63	13.49
	0/90	13.33	12.88	12.66	12.65	13.32	13.34	12.63	13.49
0.1	90/0	20.20	17.37	15.91	15.90	22.63	22.67	17.51	16.09
	0/90	13.98	14.68	15.82	15.90	12.98	12.99	17.49	16.12
0.143	90/0	24.97	21.32	19.27	19.24	28.48	28.53		22.38
	0/90	16.41	17.54	19.23	19.24	15.13	15.12		19.48
0.2	90/0	32.69	28.26	25.49	25.45	37.12	37.20	32.06	27.72
	0/90	21.45	23.10	25.05	25.43	19.71	19.74	32.17	25.88
0.3	90/0	49.65	44.57	40.81	40.65	54.72	54.93	56.28	47.10
	0/90	34.76	37.22	39.58	40.61	32.43	32.27	56.62	42.06
0.33	90/0	54.94	51.01	46.98	46.76	61.27	61.56		54.70
	0/90	40.31	42.97	45.48	46.70	37.77	37.55		48.71
0.4	90/0	70.31	65.16	60.69	60.31	73.84	75.87	73.67	69.35
	0/90	52.87	55.82	58.36	60.23	48.52	49.58	73.64	63.57
0.5	90/0	79.67	80.83	73.52	73.49	88.19	88.11	85.76	89.16
	0/90	67.16	67.02	71.73	73.47	63.06	62.97	85.74	74.76

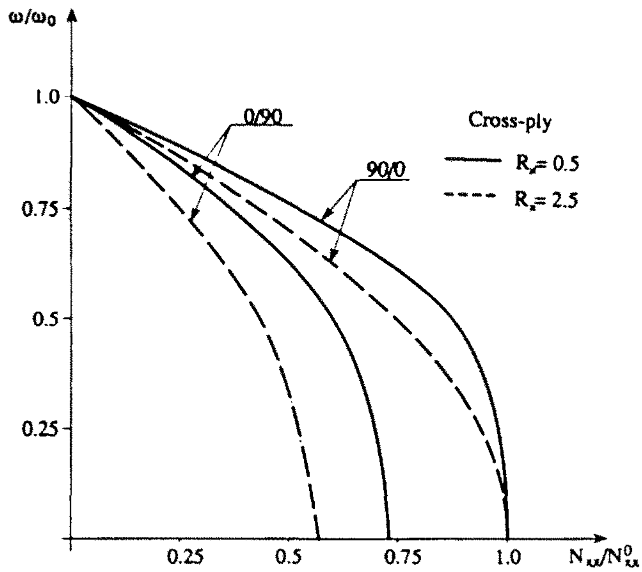

Fig. 3. Load–frequency interaction curves for cross-ply laminate.

of this effect, as well as to his use of a constant prebuckling Airy function, i.e. $\xi_f = \xi_w = 0$. As for the slightly larger but still moderate difference obtained by using COSMOS7 code (Lashkari, 1984) it is attributable to the limitation of the classical general purpose finite element code in satisfying the natural out-of-plane boundary conditions.

The effect of the usually unconsidered ξ_f and ξ_w parameters is shown by Table 1 to be quite pronounced; they yield a stiffer behavior for (90,0) layup and a more flexible one for (0,90). For the cross-ply with aspect ratio 1, the RBS results (obtained by setting $b_{ij} = 0$ in the K_{fw} matrix) are seen to be in very good agreement with the complete analysis.

Vibration analysis was also carried out for this case and the results are summarized in Fig. 3 (ω_0 is the frequency of the unloaded curved panel, ω the frequency of the loaded panel under axial compression N_{xx}, N^0_{xx} the buckling load of layup (90,0)). Here, again, a large difference is observed between (90,0) and (0,90).

(b) *Angle-ply laminate.* A comparison between the complete and the RBS analyses was carried out for buckling and vibration response. The laminate parameter b_{61}–$2b_{26}$ [see Reissner and Stavsky (1961)] comes into play in the RBS method. Its effect is mostly pronounced under inplane shear loading. The shear buckling load versus the angle-ply layup ($\pm\theta$) is plotted in Fig. 4 for radii of curvature $R_x = 0.5$ m and $R_x = 2.5$ m. With the laminate parameter not taken into account in the RBS analysis, the error introduced is minimal near $\pm 45°$ and maximal near $0°$ and $90°$. It is seen that the RBS method is curvature-sensitive, yielding a more flexible behavior for $R_x = 0.5$ m and stiffer behavior for $R_x = 2.5$ m. The interaction curves of the frequencies with the in-plane shear loading are given in Figs 5–7 for $\theta = \pm 15°$, $\theta = \pm 30°$ and $\theta = \pm 75°$ respectively, ω_0 denoting the frequency of the unloaded panel and N^0_{xy} the shear buckling load obtained from the complete analysis. For an unloaded panel, the RBS method yields very good agreement with the complete analysis; the higher the shear load level, the larger the error—again due to the prebuckling state which is not taken into consideration [through eqn (17)] in the RBS analysis.

CONCLUSIONS

The Hu–Washizu mixed formulation for the potential energy is used in buckling and vibration analyses of an arbitrary curved laminated panel. The equations are expressed in terms of the transverse displacement and the Airy stress function. The solution procedure

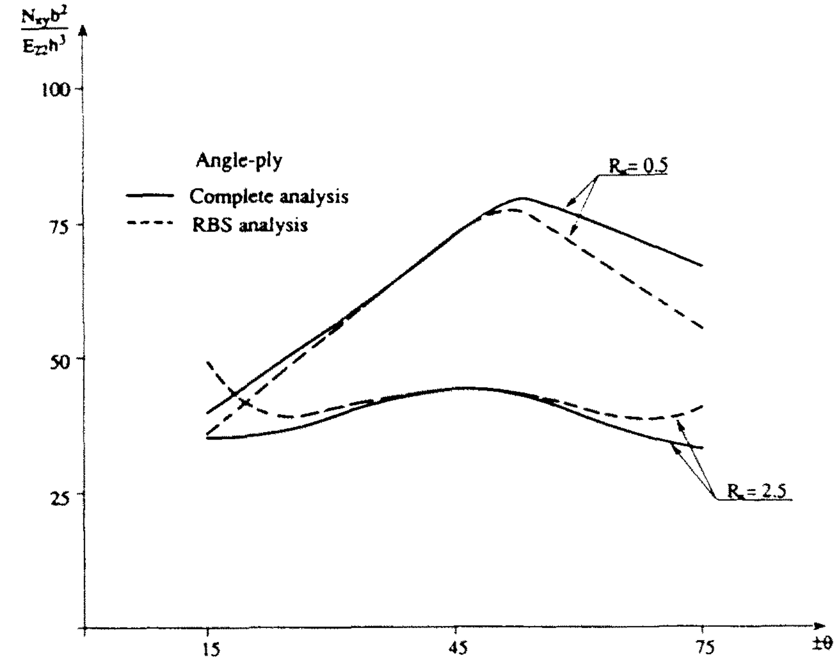

Fig. 4. Shear buckling load of angle-ply laminate: comparison of RBS method with complete analysis.

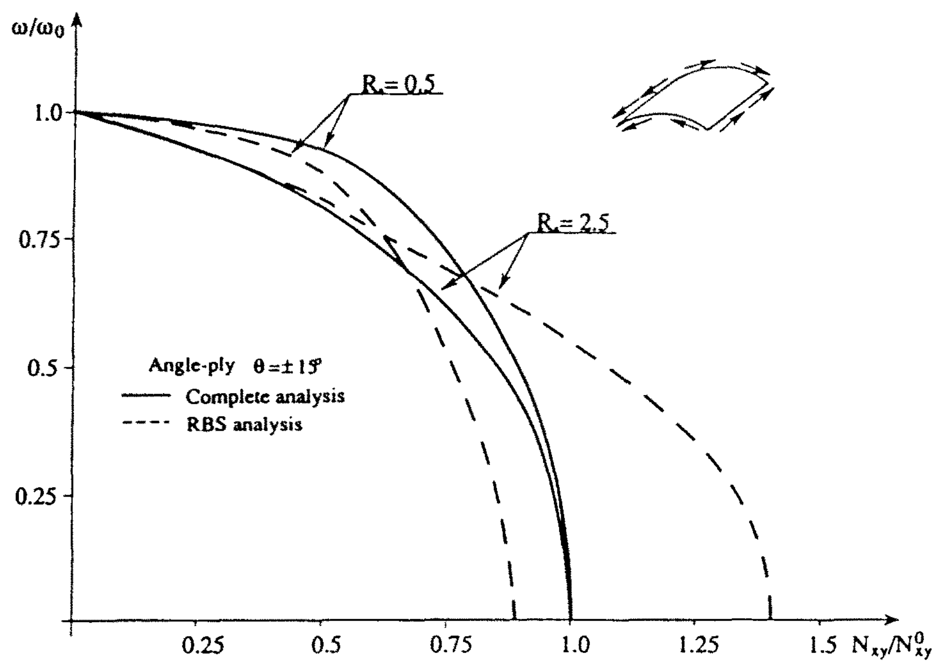

Fig. 5. Load frequency interaction curves for angle-ply $\theta = \pm 15°$.

is based on separation of the panel variables, using the eigenfunctions of an isotropic beam in both the longitudinal and lateral directions.

The paper compares a complete analysis, covering the prebuckling state, with the approximate RBS method. It was found that, in contrast to unloaded vibration response, the buckling behavior is strongly affected by the nonmembrane prebuckling stresses.

Compliance with the natural out-of-plane boundary conditions in the curved panel makes for a large difference between the (90,0) and (0,90) layups. In addition, assumption of a nonconstant Airy stress function during buckling, yields much more accurate results.

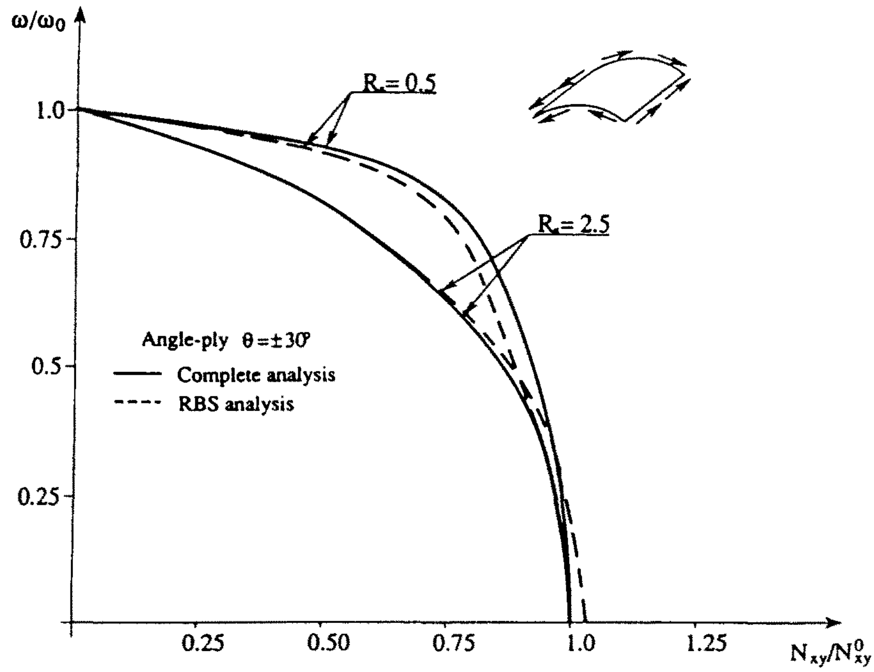

Fig. 6. Load–frequency interaction curves for angle-ply $\theta = \pm 30°$.

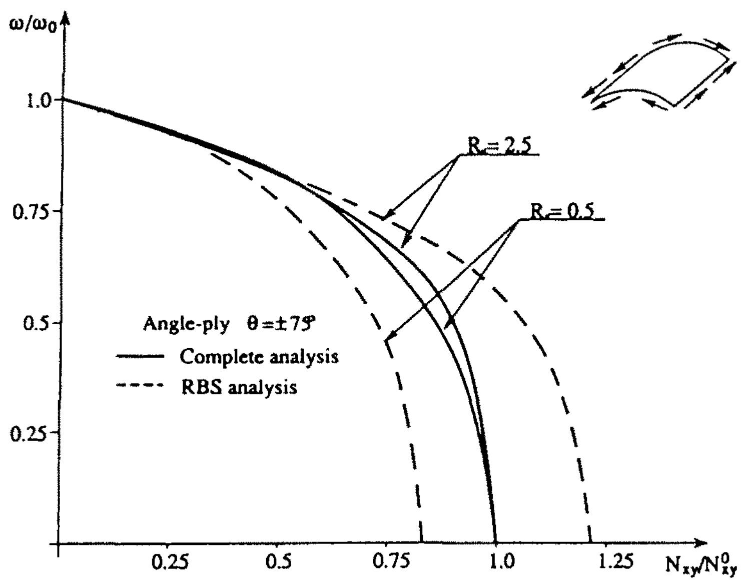

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Fig. 7. Load–frequency interaction curves for angle-ply $\theta = \pm 75°$.

The approximate RBS method is curvature sensitive and, as in the case of a flat plate, depends on the laminate parameter $(b_{61}-2b_{26})$.

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